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A class of new exact solutions in general relativity

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Abstract. A class of new exact solutions is obtained for spherically symmetric and static configurations by considering a simple relation $e^{\nu} \propto (1+x)^n$. For each integral value of n the field equations can be solved exactly and one gets a new exact solution. For physical relevance of the solutions, the pressure and the density should be finite and positive and the density, P/ρ and $dP/d\rho$ should decrease as one goes outwards from the centre to the surface of the structure. Most of the exact solutions known at present are irregular in this respect. The new exact solutions for $n = 3$, 4 and 5 are regular in this respect for a certain range of values of u(= mass/radius). The cases corresponding to $n = 1$ and 2 are already available in the literature, being obtained by other methods. For regular solutions with $dP/d\rho \leqslant 1$, the maximum values of the surface and central redshifts are 0.635 and 1.614 respectively. If one assumes the surface density to be $2\times10^{14}\ \mathrm{g\ cm^{-3}}$, a neutron star model corresponding to a mass up to 4.2 $M_{\odot}$ can be obtained. This is an upper limit for a neutron star model based upon exact solutions with completely regular behaviour and $dP/d\rho \leqslant 1$. In the limiting case when $dP/d\rho$ is infinite, the surface and the central redshifts are 1.14 and 7.36 respectively. The variation of density is slow, and for a completely regular solution the maximum value for the ratio of the central to surface densities, that is ρ_0/ρ_s, is 3.0.

1. Introduction

It is difficult to obtain explicit solutions of Einstein's gravitational field equations in terms of known analytic functions, on account of their complicated and nonlinear character. Various exact solutions of Einstein's field equations have been discussed by Kramer *et al* (1981). The first exact solutions of field equations for a perfect fluid sphere of constant density were obtained by Schwarzschild (1916). Tolman (1939) gave five new exact solutions for the fluid spheres. Of these, the III solution was the same as that given by Schwarzschild. The V and VI solutions belonged to infinite density and infinite pressure at the centre and thus cannot be considered to be of much physical relevance. In the III solution the density is constant and hence $dP/d\rho = -\infty$, thus making the expression for the speed of sound, $v_s = (dP/d\rho)^{1/2} = \sqrt{-\infty}$. Thus only the IV and VII solutions of Tolman are of physical relevance. The VII solution involves complex expressions and lacks the simplicity and elegance of an exact solution. The details of the VII solution have been worked out by Durgapal and Rawat (1980).

Tolman's IV solution is physically relevant and contains simple algebraic expressions for pressure, density, ν and λ. In particular, the expression for ν is so simple that one can work out the trajectories of photons and neutrinos with ease (Durgapal and Pande 1979). But this solution has its own limitations, which we may summarise as follows (Durgapal and Pande 1980).

(a) The maximum value of the ratio of the mass and the radius, that is $u = m/a$, is $\frac{1}{3}$ and for this value of u, the pressure and the density become infinite at the centre.

(b) For $0.25 < u \leqslant \frac{1}{3}$, the value of P/ρ increases with increasing values of r and attains a maximum at a certain value of r within the configuration. Thus the ratio P/ρ is not a maximum at the centre but at some other point. This restricts the use of this solution to the values of $u \leqslant 0.25$.

(c) The speed of sound, $v_s = (dP/d\rho)^{1/2}$, is 0.447 at the centre but at some other point it is a maximum. For all values of u, the value of $dP/d\rho$ increases as we move outwards from the centre. However, this is not a very serious drawback because Bondi (1964) has discussed the solutions in which $dP/d\rho$ jumps from $\frac{1}{3}$ to 1 at the boundary. But this is not consistent with the different equations of state known for nuclear matter (Canuto 1975).

In this paper, a method is given for treating the nonlinear differential equations applying to gravitational equilibrium of perfect fluids, in such a manner as to obtain a class of exact solutions which may have measure of physical interest. Some of the particular cases of this new class of exact solutions have been found to be free from any of the drawbacks discussed above. The IV solution of Tolman is a particular case of this new class of solutions.

The general assumptions made for solving Einstein's field equations are the same as those given by Bondi (1964). The solutions are continuous at the boundary with the external Schwarzschild solutions, that is at the boundary, $r = a$, we have

$$P(r = a) = 0, \qquad e^{\nu(a)} = e^{-\lambda(a)} = 1 - 2m/a = 1 - 2u.$$

The pressure and density must follow one or both of the following restrictions.

(i) The trace of the energy–momentum tensor is positive, that is, $P \leqslant \rho c^2/3$.

(ii) The signal cannot propagate at a velocity greater than that of light, that is, $dP/d\rho \leqslant c^2$ (Zeldovich 1961).

Taking the velocity of light $c = 1$ and the gravitational constant $G = 1$, the relations between the density ρ, the pressure P and the energy–momentum tensor of a perfect fluid are given by

$$\rho = T_0^0, \qquad P = -T_1^1 = -T_2^2 = -T_3^3. \tag{1}$$

2. Field equations and their solutions

2.1. Field equations

The line element is given by

$$\begin{aligned} &ds^2 = g_{00}dt^2 + g_{kl}\, dx^k\, dx^l, && && \text{where } k, l = 1, 2, 3, \\ &g_{00} = e^{\nu(r)}, && g_{11} = -e^{\lambda(r)}, && g_{22} = -r^2, \\ &g_{33} = -r^2 \sin^2\theta, && g_{kl} = 0 && \text{for } k \neq l. \end{aligned} \tag{2}$$

Here ν and λ are functions of r alone. The resulting field equations are

$$-8\pi T_1^1 = 8\pi P = e^{-\lambda}(\nu'/r + 1/r^2) - 1/r^2, \tag{3}$$

$$-8\pi T_2^2 = -8\pi T_3^3 = 8\pi P = e^{-\lambda}[\tfrac{1}{2}\nu'' + \tfrac{1}{4}\nu'^2 - \tfrac{1}{4}\nu'\lambda' + (\nu' - \lambda')/2r], \tag{4}$$

$$-8\pi T_0^0 = -8\pi\rho = -1/r^2 - e^{-\lambda}(\lambda'/r - 1/r^2). \tag{5}$$

Equations (3) and (4) give

$$-e^{-\lambda}\lambda'(\nu'/4+1/2r)+e^{-\lambda}(\nu''/2+\nu'^2/4-\nu'/2r-1/r^2)+1/r^2. \tag{6}$$

Let us now assume that the value of ν is given by a general expression

$$e^{\nu}=A(1+Cr^2)^n, \qquad \text{where } n \text{ is a parameter.} \tag{7}$$

We have chosen this simple expression because this gives us a very simple relation for the redshift from any region of the configuration. Further, a simple expression for e^{ν} can be helpful in calculating the trajectories of ultra-relativistic particles in the gravitational field.

Substitution of equation (7) into equations (3)–(6) leads to

$$8\pi P/C=2nZ/(1+x)+(Z-1)/x, \tag{8}$$

$$8\pi\rho/C=(1-Z)/x-2\,dZ/dx, \tag{9}$$

$$dZ/dx+Q(x)Z=f(x),$$

where

$$Q(x)=-[1+2x+(1+2n-n^2)x^2]/x(1+x)[1+(n+1)x], \tag{11}$$

$$x=Cr^2, \qquad e^{-\lambda}=Z, \qquad f(x)=-(1+x)/x[1+(n+1)x]. \tag{12}$$

The solution of equation (10) is given by

$$Z=FK-FI, \tag{13}$$

where

$$F=\exp\left(-\int Q(x)\,dx\right)=x/(1+x)^{n-2}[1+(n+1)x]^{2/(n+1)} \tag{14}$$

and

$$I=\int(1+x)^{n-1}\,dx/x^2[1+(n+1)x]^{(n-1)/(n+1)}. \tag{15}$$

This integral can be solved very easily for any value of n, because

$$I=\int\left(\frac{1}{x^2}+\frac{(n-1)}{x}+\frac{(n-1)(n-2)}{2}+\frac{(n-1)(n-2)(n-3)x}{3}+\cdots\right)$$

$$\times dx[1+(n+1)x]^{-(n-1)/(n+1)}. \tag{16}$$

The first two terms give

$$\int(x^{-2}+(n-1)/x)\,dx[1+(n+1)x]^{-(n-1)/(n+1)}=-x^{-1}[1+(n+1)x]^{2/(n+1)}. \tag{17}$$

Other terms in the expansion (16) can be evaluated for different values of n. With the values of e^{ν} and $e^{-\lambda}=Z$ known, we can calculate the pressure, density, the ratio P/ρ and the value of $dP/d\rho$ at any point within the configuration. The constants A, C and K can be determined by considering the boundary conditions $P(r=a)=0$, $e^{-\lambda(a)}=e^{\nu(a)}=1-2u$. From the condition $P(a)=0$ we get $x_1=Ca^2=u/[n-(2n+1)u]$, and from $e^{\nu(a)}=1-2u$ we get

$$A=[1-(2n+1)u/n]^n/(1-2u)^{n-1}.$$

The value of the constant K appearing in the expression for $e^{-\lambda}=Z=FK-FI$ is obtained after writing the expression for $e^{-\lambda}$ for a particular value of n.

3. Solutions for different values of n

By taking different values of n, we can obtain a very large number of exact solutions of Einstein's field equations. Here, we have given the exact solutions for $n = 1, 2, 3, 4$ and 5 and discussed the physical relevance of these solutions. The calculations for higher values of n can be done by evaluating the integral I.

3.1. $n=1$

The solutions are given by

$$\begin{aligned} &e^{-\lambda} = (1+x)(1+Kx)/(1+2x), \qquad e^{\nu} = A(1+x), \\ &\frac{8\pi P}{C} = \frac{(1+K+3Kx)}{(1+2x)}, \\ &\frac{8\pi\rho}{C} = \frac{1-3K-3Kx}{1+2x} + \frac{2(1+Kx)}{(1+2x)^2}, \end{aligned} \tag{18}$$

where

$$A = 1-3u, \qquad x_1 = Ca^2 = u/(1-3u), \qquad K = -(1-3u). \tag{19}$$

On substituting the values of various constants in the expressions for P, ρ, ν and λ it can be seen that the results are identical to Tolman's IV solutions. The limitations of the IV solution have already been discussed.

3.2. $n=2$

The solutions can be written as

$$\begin{aligned} &e^{-\lambda} = 1 + Kx/(1+3x)^{2/3}, \qquad e^{\nu} = A(1+x)^2, \\ &8\pi P/C = [4 + K(1+5x)/(1+3x)^{2/3}]/(1+x), \\ &8\pi\rho/C = -K(3+5x)/(1+3x)^{5/3}, \end{aligned} \tag{20}$$

where

$$\begin{aligned} &A = (1-2.5u)^2/(1-2u), \qquad x_1 = Ca^2 = u/(2-5u), \\ &K = -(2-2u)^{2/3}(2-5u)^{1/3}, \\ &dP/d\rho = [1 - 5x^2 - (2/K)(1+3x)^{5/3}](1+3x)/5(1+x)^3. \end{aligned}$$

The value of P/ρ is a maximum at the centre and decreases with increasing values of r. But

$$\begin{aligned} \frac{d}{dx}(dP/d\rho) &= -\tfrac{2}{5}[4x(2+5x) + (5-x)(1+3x)^{5/3}/K](1+x)^{-4} \\ &= -2/K \text{ (at the centre, } r=0) = \text{positive}. \end{aligned}$$

Because K is always negative the nature of $dP/d\rho$ is irregular near the centre for any value of u. The results do not show any significant advantage over Tolman's IV solution. On substituting the values of constants in terms of u, the solution becomes identical to that obtained by Kuchowicz (1975), Adler (1974) and Adams and Cohen (1975).

3.3. $n=3$

The solutions are given by

$$e^{-\lambda}=\frac{2-x}{2(1+x)}+\frac{Kx}{(1+x)(1+4x)^{1/2}}, \qquad e^{\nu}=A(1+x)^3,$$
$$8\pi P/C=[\tfrac{9}{2}(1-x)+K(1+7x)(1+4x)^{-1/2}](1+x)^{-2},$$
$$8\pi\rho/C=[\tfrac{3}{2}(3+x)-3K(1+3x)(1+4x)^{-3/2}](1+x)^{-2}, \qquad (21)$$
$$\frac{dP}{d\rho}=\frac{9-3x-2K(1-x-14x^2)(1+4x)^{-3/2}}{5+x-2K(5+23x+30x^2)(1+4x)^{-5/2}},$$

where

$$A=(1-7u/3)^3/(1-2u)^2, \qquad x_1=Ca^2=u/(3-7u),$$
$$K=-9(1-x_1)(1+4x_1)^{1/2}/2(1+7x_1).$$

P/ρ decreases with increasing values of r for all values of u and

$$P_0/\rho_0=(9+2K)/(9-6K) \qquad (dP/d\rho)_0=(9-2K)/(5-10K).$$

For $K=-1.5$, the value of P/ρ at the centre (that is P_0/ρ_0) is $\frac{1}{3}$ and $u=0.292$. But the nature of $dP/d\rho$ is not regular for every value of u. In table 1, the values of P_0/ρ_0, the maximum value of $dP/d\rho$ and the position of this maximum value, x_1, the constant K and the mass of the neutron star in solar mass units (the mass has been calculated by assuming the density at the surface, ρ_s, to be equal to 2×10^{14} g cm^{-3}) have been given for different values of u.

Table 1. Various parameters for the solution with $n=3$.

u	P_0/ρ_0	$(dP/d\rho)_{max}$	Position (r/a) of $(dP/d\rho)_{max}$	K	$x_1=Ca^2$	$m/M_\odot$	ρ_0/ρ_s
0.01	0.005	0.3644	1.0	−4.410	0.0034	0.027	1.014
0.10	0.061	0.4140	1.0	−3.575	0.0435	0.766	1.176
0.15	0.103	0.4531	1.0	−3.088	0.0769	1.315	1.304
0.20	0.158	0.5054	1.0	−2.572	0.1250	1.889	1.478
0.25	0.236	0.5828	0.9	−2.012	0.2000	2.494	1.724
0.30	0.362	0.7207	0.7	−1.375	$\frac{1}{3}$	3.228	2.096
0.35	0.635	1.0629	0.5	−0.565	0.6363	4.334	4.473
0.375	1.000	1.8000	0.0	0.000	1.0000	5.104	3.000
0.3908	$\frac{5}{3}$	infinity	0.0	0.500	1.4781	5.312	2.942

The value of $dP/d\rho$ is less than one throughout the configuration for $u<0.35$, but the behaviour of $dP/d\rho$ is not regular ($dP/d\rho$ is not a maximum at the centre). For $K\geqslant0$, that is $u\geqslant0.375$, the behaviour of $dP/d\rho$ is regular. One important feature at $u=0.375$ is that $dP/d\rho\geqslant1$ throughout the structure. At the surface its value is one and at the centre its value is 1.8. In this particular case, if we consider the speed of sound to be supraluminal for nuclear matter (that is, $dP/d\rho\geqslant1$ for $\rho\geqslant2\times10^{14}$ g cm^{-3}), we get a mass of 5.1 $M_\odot$. For this value of u, the surface redshift is 1.00 and the central redshift is 4.75. For $K=0.5$ and $u=0.3908$, the value of $dP/d\rho$

becomes infinite at the centre and P_0/ρ_0 is $\frac{5}{3}$. For this extreme case, the surface and central redshifts are respectively 1.142 and 7.356. For $u > 0.3908$, the value of $\mathrm{d}P/\mathrm{d}\rho$ becomes negative and thus the solutions are physically irrelevant beyond this value of u.

3.4. $n=4$

The solutions are given by

$$\mathrm{e}^{-\lambda} = \frac{7-10x-x^2}{7(1+x)^2} + \frac{Kx}{(1+x)^2(1+5x)^{2/5}}, \qquad \mathrm{e}^{\nu} = A(1+x)^4,$$
$$8\pi P/C = [\tfrac{16}{7}(2-7x-x^2) + K(1+9x)(1+5x)^{-2/5}]/(1+x)^3, \tag{22}$$
$$8\pi\rho/C = [\tfrac{8}{7}(9+2x+x^2) - K(3+10x-9x^2)(1+5x)^{-7/5}]/(1+x)^3,$$
$$\frac{\mathrm{d}P}{\mathrm{d}\rho} = \frac{4(13-12x-x^2)-7K(1-2x-27x^2)(1+5x)^{-7/5}}{2(25+2x+x^2)-7K(5+31x+47x^2-27x^3)(1+5x)^{-12/5}},$$

where

$$A = (1-2.25u)^4/(1-2u)^3, \qquad x_1 = Ca^2 = u/(4-9u),$$
$$K = -16(2-7x_1-x_1^2)(1+5x_1)^{2/5}/7(1+9x_1).$$

P/ρ is regular for this solution as its value is a maximum at the centre and decreases with increasing values of r. At the centre we have

$$P_0/\rho_0 = (32+7K)/(72-21K) \qquad \text{and} \qquad (\mathrm{d}P/\mathrm{d}\rho)_0 = (52-7K)/(50-35K).$$

For $K = -\frac{4}{7}$, $u = 0.287$ the value of P_0/ρ_0 is $\frac{1}{3}$. For $u < 0.30$, the value of $\mathrm{d}P/\mathrm{d}\rho < 1$ throughout the configuration but its maximum is not at the centre. For $0.30 \leqslant u \leqslant 0.313$, the value of $\mathrm{d}P/\mathrm{d}\rho \leqslant 1$ and its behaviour is regular ($\mathrm{d}P/\mathrm{d}\rho$ is a maximum at the centre and decreases with increasing values of r). For $u = 0.313$, $(\mathrm{d}P/\mathrm{d}\rho)_0 = 1$, the surface redshift is 0.635, the central redshift is 1.614 and mass of the neutron star comes out to be $4.19\,M_\odot$ (assuming $\rho_s = 2\times10^{14}\ \mathrm{g\,cm^{-3}}$, Durgapal *et al* 1979) which is consistent with the values obtained by others. For $K = \frac{10}{7}$ and $u = 0.3705$, $P_0/\rho_0 = 1$ and $(\mathrm{d}P/\mathrm{d}\rho)_0$ is infinite. For this extreme case the surface and central redshifts are 0.965 and 3.754 respectively. For all $u \geqslant 0.30$ the value of $\mathrm{d}P/\mathrm{d}\rho$ is a maximum at the centre and decreases with increasing r.

3.5. $n=5$

The solutions can be written as:

$$\mathrm{e}^{-\lambda} = [1 - x(309+54x+8x^2)/112 + Kx/(1+6x)^{1/3}]/(1+x)^3, \qquad \mathrm{e}^{\nu} = A(1+x)^5,$$
$$8\pi P/C = [(475-4125x-1050x^2-200x^3)/112 + K(1+11x)/(1+6x)^{1/3}](1+x)^{-4}, \tag{23}$$

$$8\pi\rho/C = [(1935+15x+450x^2+120x^3)/112$$
$$- K(3+11x-22x^2)(1+6x)^{-4/3}](1+x)^{-4},$$

$$\frac{dP}{d\rho}=\frac{5(241-411x-60x^2-8x^3)-112K(1-3x-44x^2)(1+6x)^{-4/3}}{3(515-57x+36x^2+8x^3)-112K(5+39x+66x^2-88x^3)(1+6x)^{-7/3}},$$

where

$$A=(1-2.2u)^5/(1-2u)^4, \qquad x_1=Ca^2=u/(5-11u),$$

and

$$K=-(475-4125x_1-1050x_1^2-200x_1^3)(1+6x_1)^{1/3}/112(1+11x_1).$$

For all values of u, the value of P/ρ is a maximum at the centre and decreases with increasing values of r. For $u\leqslant 0.265$, the value of $dP/d\rho$ is a maximum at the centre and then decreases with increasing values of r. For $u=0.265$, $(dP/d\rho)_0=0.85$. For $0.265<u\leqslant 0.29$, the value of $dP/d\rho$ is not a maximum at the centre. For $u>0.29$, the nature of $dP/d\rho$ becomes erratic, being negative at some values of r within the configuration. However, this solution gives us the widest range of applicability with a regular behaviour of both P/ρ and $dP/d\rho$. For $u=0.265$, the surface redshift is 0.46 and the central redshift is 1.48. The neutron star mass for this value of u comes to be 3.387 $M_\odot$. The value of P_0/ρ_0, $x_1=Ca^2$, the constant K, the maximum value of $dP/d\rho$ and its position and the mass of the neutron star have been shown in table 3.

Table 2. Various parameters for the solution with $n=4$.

u	P_0/ρ_0	$(dP/d\rho)_{max}$	Position (r/a) of $(dP/d\rho)_{max}$	K	$x_1=Ca^2$	$m/M_\odot$	ρ_0/ρ_s
0.01	0.005	0.4045	1.0	−4.451	0.0026	0.027	1.012
0.10	0.061	0.4550	0.8	−3.335	0.0323	0.837	1.155
0.15	0.103	0.4968	0.7	−2.678	0.0660	1.510	1.264
0.20	0.160	0.5580	0.6	−1.979	0.0909	2.277	1.405
0.25	0.241	0.6587	0.4	−1.215	0.1429	3.106	1.592
0.30	0.375	0.8806	0.0	−0.335	0.2308	3.965	1.842
0.313 141	0.429	1.000	0.0	−0.071	0.2649	4.191	1.918
0.35	0.681	2.106	0.0	0.799	0.4118	4.818	2.117
0.370 483	1.000	infinity	0.0	$\frac{10}{7}$	0.5566	5.157	2.129

Table 3. Various parameters for the solution with $n=5$.

u	P_0/ρ_0	$(dP/d\rho)_{max}$	Position (r/a) of $(dP/d\rho)_{max}$	K	$x_1=Ca^2$	$m/M_\odot$	ρ_0/ρ_s
0.01	0.005	0.4336	0.0	−4.091	0.0020	0.027	1.011
0.10	0.061	0.4935	0.0	−2.692	0.0256	0.838	1.143
0.15	0.104	0.5459	0.0	−1.866	0.0448	1.515	1.257
0.20	0.161	0.6274	0.0	−0.985	0.0714	2.288	1.363
0.25	0.244	0.7765	0.0	−0.163	$\frac{1}{9}$	3.127	1.520
0.265	0.277	0.8506	0.0	0.300	0.1271	3.387	1.574

Note: The mass of the neutron star $m/M_\odot$ has been calculated by assuming the density at $r=a$, $\rho_s=2\times10^{14}\ \mathrm{g\ cm^{-3}}$.

4. Discussion

The new class of exact solutions can provide us with a large number of solutions of Einstein's field equations by selecting different values of the parameter n. Pressure, density, ν, λ and $\mathrm{d}P/\mathrm{d}\rho$ can be expressed in terms of simple algebraic expressions.

For $n=1$, the solution is identical to Tolman's IV solution, and for $n=2$, we get the solution discussed by Adler (1974). Both these solutions are irregular in the behaviour of $\mathrm{d}P/\mathrm{d}\rho$. Hence they are not suitable for application to a neutron star model because the equations of state for nuclear matter show a regular behaviour of $\mathrm{d}P/\mathrm{d}\rho$. *The solutions for which* $\mathrm{d}P/\mathrm{d}\rho$ *increases with increasing density are termed regular in this paper.*

The physical nature of the solutions corresponding to $n=3$, 4 and 5 is shown with the help of tables 1, 2 and 3. The maximum value of u for which the solutions are regular and $\mathrm{d}P/\mathrm{d}\rho \leqslant 1$ is found to be 0.313 (case $n=4$). The surface and central redshifts for this case are respectively 0.635 and 1.614. Assuming the surface density to be $2\times10^{14}\ \mathrm{g\,cm^{-3}}$, the mass of the corresponding neutron star model comes out to be $4.19\ M_\odot$. However, the maximum values of the surface and the central redshifts are 1.14 and 7.36 respectively (when $\mathrm{d}P/\mathrm{d}\rho=\infty$ for $n=3$ and $u=0.3908$). For regular solutions, the condition $P\leqslant\rho/3$ is not satisfied for $n\leqslant4$. For $n=5$ all the regular solutions satisfy both the restrictions (i) $P\leqslant\rho/3$ and (ii) $\mathrm{d}P/\mathrm{d}\rho\leqslant1$. The maximum mass for the corresponding neutron star model comes out to be $3.387\ M_\odot$.

For all these solutions, the density decreases as we go outwards from the centre. The variation of density is slow for all the regular solutions. For $n=3$, the maximum value for the ratio of central to surface densities, that is ρ_0/ρ_s, is 4.5. For $n=4$, when $\mathrm{d}P/\mathrm{d}\rho=1$ at the centre and the solution is regular, we have $\rho_0/\rho_s=1.918$. Even when $\mathrm{d}P/\mathrm{d}\rho$ is infinite, ρ_0/ρ_s is 2.129. For $n=5$, the maximum value of ρ_0/ρ_s for regular solutions is 1.574.

In these solutions the expression for $\mathrm{e}^{\nu}=A(1+x)^n$ is simple and one can calculate the trajectory of ultra-relativistic particles with comparative ease. The solutions for the values of $n>5$ can also be worked out and their physical relevance can be seen.

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